Life-Cycle Portfolio Choice in an Intergenerational Set-up

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Abstract This study solves the life-cycle portfolio choice problem of an altruistic father valuing the human-capital of his offspring and facing competing challenges of his consumption needs during working life and retirement. The problem is solved using dynamic programming where life-cycle profiles are generated using simulations. This study contributes to the financial literature on risk, borrowing constraints and portfolio choice by introducing inter-generational dynamics with children's consumption and human-capital as investment options for the parent.

Keywords Portfolio Choice · Human capital
JEL classification: D60; I26;

1 Literature Review

The seminal work of Samuelson (1969) and Merton (1969) addressed the choice of portfolio problem in a multi-period, continuous time framework and showed that portfolio allocation is not a function of wealth and age. Assuming i.i.d returns, CRRA utility and frictionless markets, over the lifetime, all agents should hold a positive fraction of risky asset (stocks), the share of which should remain constant (see Ameriks & Zeldes (2004)).

Merton (1970) introduced a separate labour income process to the above formulation under the assumption of completely tradable labour and his conclusions were qualitatively the same.

Bodie et al. (1992) endogenized the labour supply by including leisure into the utility function. They assumed a set-up with fixed retirement age and borrowing against future labour income. Their conclusion was that when future labour income is certain, it is optimal for employed investors to have a higher fraction of the risky asset in their portfolio than retired investors. That is, working individuals should tilt their portfolio towards the risky asset when they have the option of calibrating their labour supply post shocks.

Cocco et al. (2005) argued that since, households cannot borrow against their future labour incomes due to moral hazard problems, they face borrowing constraints in the early phases of their lives. Also, due to the fact that there are no well developed insurance market for labour income they also face labour income risk. Hence, they model a finite life span investor facing labour income risk, mortality risk, short-sale and borrowing constraints. Working with a riskless and risky asset and assuming that the risky asset returns are correlated with labour income shocks. In contrast to Samuelson and Merton, they find that the optimal fraction of risky asset in the portfolio goes down with age and wealth.

Along similar lines, Koo (2002) and Viceira (2001) study an infinite horizon model with uninsurable labour risk and

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portfolio choice. Viceira (2001) considers a finite probability every period of zero future labour income forever, in order to capture retirement effects. Models of infinite-horizon,however, are not well-suited for life-cycle scenarios due to their stationary nature. The accumulated wealth to future labour income ratio, on which the portfolio composition depends, is by nature non-stationary.

There is related literature, along similar lines, dealing with finite horizon models (Cocco et al. (2005), Bertaut & Haliassos (1997), Davis & Willen (2000), Gakidis (1998)), taxes (Dammon et al. (2001)), asset pricing implications (Constantinidies et al. (1998) , Storesletten et al. (2007)) and housing (Cocco (2005), Hu (2005), Yao & Zhang (2005)).

It is worth mentioning that all the above studies consider the life-cycle portfolio allocation problem from a single working individual's perspective. However, in reality the working individual would be,in most cases, a part of a household.

A working parent's preferences, apart from his own lifetime consumption, would also include his offspring's lifetime consumption if he is altruistic towards his offspring. Even when he is not altruistic, the parent might invest in his offspring's human capital as a selfish way of transferring his resources inter-temporally. One then would be inclined to believe that inter-generational dynamics, would lead to different inter-temporal outcomes than in pure life-cycle models. As Becker (1991) pointed out, a household is a kind of an enterprise with its own complex sets of preferences, output and constraints.

The review of the literature suggests that while most of the finance literature has focussed on life-cycle portfolio choice in the presence of borrowing constraints, risky labour income, endogenous leisure or housing not much work has explored the intergenerational dynamics with children's human capital as an investment option for the parent either due to altruistic or pure exchange reasons. Similarly, while there exists significant economics literature on the intergenerational set-up and human capital acquisition, a more realistic life-cycle savings decision problem of a parent facing the competing requirements of consumption, children's education and retirement savings under uncertain wages and financial asset returns, bequests and borrowing constraints has not been attempted.

While the literature review has mainly focussed on life-cycle models, relevant empirical literature will get cited with the results as the discussion moves ahead.

2 A Two-Generation, Discrete Time, Life-Cycle Model

2.1 The Environment

The model assumes a single parent (father) with a single offspring (son) both of whom would be jointly referred to as the household.

Time is discrete and the life-cycle model starts from time, $t = 1$, when the son is just an infant (age=1) and the father has started working (age=21).
"t" denotes absolute time in years, "age" denotes the age in years of the person in focus (either father or son). For the son, t and age will be the same. There are 70 time periods in the model.
A phase is defined as the son's phase in life: education phase and adult phase. The first 20 time periods ($1 \leq t \leq 20$) have been defined as the education phase. Adult phase starts from $t = 21$ onwards. The education phase is also when the father and son are a joint household. Hence, the terms education phase and joint phase are used interchangeably.

In the first five years (pre-school stage) there are neither education nor labour options available to the son. The only expense the father incurs on the son, during this period, is consumption expense. Even though a child consumes less than the parent, in the simulations the father-son consumption expenditures are assumed as equal since health related expenses of the child are not explicitly modelled.

From $6 \leq age < 20$ the son can either go to acquire human-capital or work in the informal child-labour market to augment the family income. (Note: A definition of a child is someone who is not an adult). The time of entry of the independent (grown-up) son into the formal labour market is denoted as $t = sw = 21$ (son working) which is also the time the son separates and starts a new family.

From $t = sw = 21$ onwards, the father and son are economically separate and have no interaction except for a last period (of father's life) bequest. At time $t = fr = 41$ the father retires (father's age=61) and the father is dead at time $t = fd = 50$ (father's age=70). He leaves a pre-determined bequest, which is a constant fraction of his last-period wealth, to his son.

The son starts his adult life with zero assets. The son retires at $t = sr = 61$ and dies at $t = sd = 70$ which is also the last period of the model.

Human-capital

From, $6 \quad t \quad 20$, the father has an option of either educating the son or sending him to the child-labour market or a combination of both. The son is endowed with a unit amount of time in each period. The fraction of time devoted to education in time-period t is denoted as l_t. The father also decides the amount of books b_t to be invested in each period. The son's future income as an adult depends on the level of human capital accumulated in the education phase ($6 \quad t \quad 20$).

The household faces borrowing constraints every period as the father cannot borrow against his son's future labour income due to moral hazard problems. The model also assumes that the father has no borrowing options against his own future labour income due to labour-income risk.

The father's human capital $H_{f;t} = H_{dad}$ is assumed constant over time. Subscript f stands for father, s stands for son.

The Ben-Porath (1967) model was used for the human capital production function. The child's human capital at period $t + 1$, in the education phase, is the function of previous period's human capital $H_{s;t}$, ability level q, time fraction spent studying l_t, books invested b_t and the rate of depreciation d

$$
\begin{aligned}
H_{f;t+1} &= H_{f;t} = H_{dad} \\
H_{s;t+1} &= f(q; l_t; H_{s;t}; b_t) \\
&= H_{s;t}(1 \quad d) + q(l_t)^{aa}(H_{s;t})^{bb}(b_t)^{cc}
\end{aligned} \tag{1}
$$

The human capital of the pre-school son is assumed to stay constant and is assumed as equal to the base level of human capital $\bar{H}$ (which is the lowest possible level of human-capital and there is no further depreciation from that). Human-capital acquisition can start from age 6 onwards.

Since, the nutritional effect on human-capital is not modelled in the pre-school stage, human-capital is assumed to remain at the base level upto age 6. Also, since the father is solving the life-cycle model at $t = 1$ and assumes the son will be a separate household once he reaches age 21, the father only takes into account formal education upto graduation. Human-capital accumulation prospects of the independent son through On-the-Job Training (OJT) is not modelled.

The maximum $H_{s;t+1}$ the child can have corresponds to $l_t = l_{max}$ and $b_t = b_{max}$.

$$
H_{s;t+1}(max) = f(q; l_{max}; H_{s;t}; b_{max}) = H_{s;t}(1 \quad d) + q(l_{max})^{aa}(H_{s;t})^{bb}(b_{max})^{cc} \tag{2}
$$

$l_{max} = 0$ when the child is in the pre-school stage and equal to 1 for other periods in the education phase.
b_{max} is the maximum investment possible in books. b_{max} is zero at pre-school and then constant for a given income category. It varies for different income categories to re ect the inaccessibility of good schools for poor households.

The father has a per-period dis-utility from not being able to afford the maximum inputs (time and books) for his child. The dis-utility is proportional to the amount of human capital foregone $f(q; l_{max}; H_{s;t}; b_{max}) \quad f(q; l_t; H_{s;t}; b_t)$.

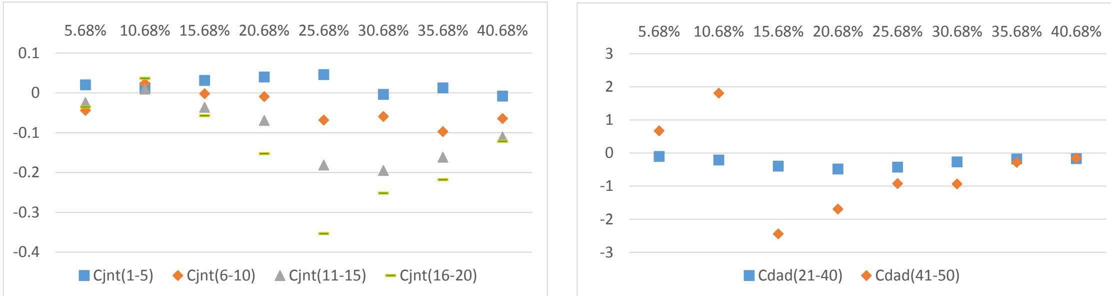
The amount of dis-utility to a father due to lower inputs, $l_t < 1$ and $b_t < b_{max}$ is :

$$
pen \quad f(q; l_{max}; H_{s;t}; b_{max}) \quad f(q; l_t; H_{s;t}; b_t) \tag{3}
$$

where pen is the amount of dis-utility per-unit of human capital foregone.

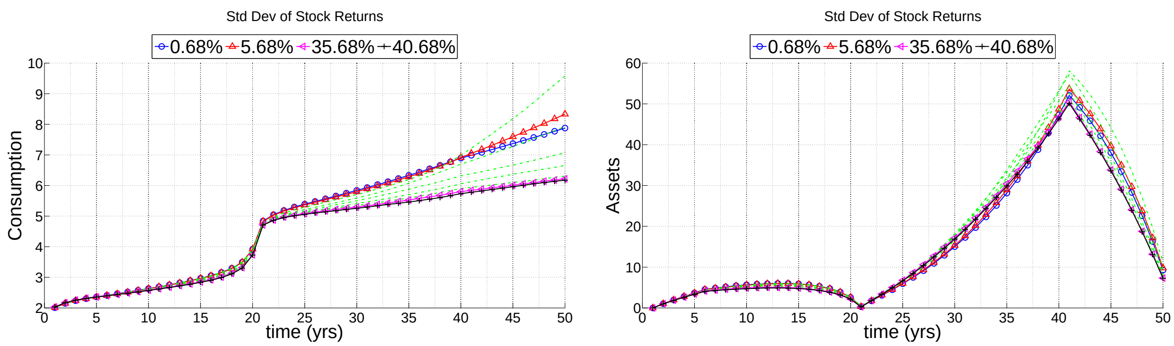
Father's wage

Wages are subjected to a per period wage shock e_t which can take two values, $e_t = \{e_{low}; e_{high}\}$ such that $E[e_t] = 1$: The actual wage of an adult with human-capital level H_{dad} at time t 0038 Tf 4m038 Tf 4m038 7.0038 Tf 6.596 -1.56.921 4.04645 Tf 2.62



(a) Joint-period consumption elasticities (b) Independent dad's consumption elasticities

Fig. 3: Graduate dad: Elasticities for standard deviations of stock returns $\in [0.68\%, 40.68\%]$



(a) Consumption (all 50 time-periods) (b) Assets

Fig. 4: Graduate dad: Life-cycle plots for standard deviations of stock returns $\in [0.68\%, 40.68\%]$

Table 5: Graduate dad: Impact of a change in std dev of stock return from benchmark of 15.68%. Std Dev $\in [0.68\%, 40.68\%]$

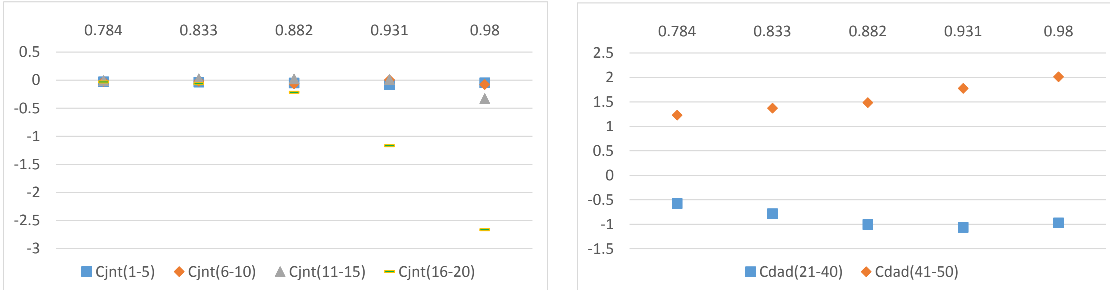
$\Delta Std.Dev$	$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$	$\Delta C_{dad(21-40,41-50)}$	ΔBeq	$\Delta Edu(yrs)$	$\Delta Books(Tot)$	ΔH_{son}
-15 %	-0.28, 00.13, 00.21, 00.19 %	03.67, 01.01 %	-0.64 %	00.00 %	00.00 %	00.00 %
-5 %	-0.14, 00.02, 00.16, 00.24 %	02.02, 13.64 %	20.78 %	00.00 %	00.00 %	00.00 %
5 %	00.18, -0.05, -0.33, -0.73 %	-2.43, -8.65 %	-10.86 %	00.00 %	00.00 %	00.00 %
15 %	00.36, -0.69, -2.19, -3.57 %	-5.76, -16.73 %	-20.44 %	00.00 %	00.00 %	00.00 %
25 %	00.36, -1.50, -3.44, -5.08 %	-7.37, -18.61 %	-22.10 %	00.00 %	00.00 %	00.00 %

$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$: % changes in per-capita consumptions (from benchmark) for time periods 1-5, 6-10, 11-15, 16-20.
$\Delta Edu(yrs)$, $\Delta Books(Tot)$: % changes in total years of education and books from benchmark values.

4.0.3 Time Discount Factor

This subsection explores the impact of discount factor on life-cycle variables. The range of discount factors considered are from 0.735 to 0.98 with the benchmark value being 0.98.

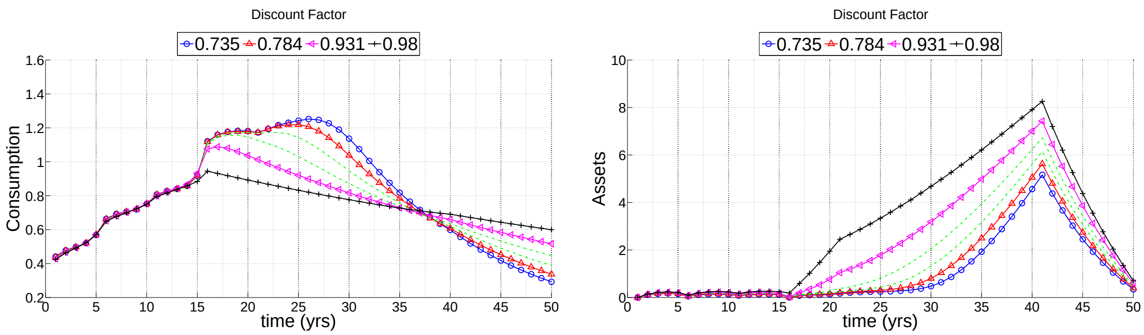
Uneducated Dad : Figure 6 shows that as discount rate increases, due to higher patience, there is a systematic fall in $C_{joint(16-20)}$ and $C_{dad(21-40)}$ and a commensurate increase in assets enabling the rise in dad's post retirement consumption. $C_{joint(1-5)}, C_{joint(6-10)}$ and $C_{joint(11-15)}$ remain mostly inelastic. Human-capital investments are not affected.



(a) Joint-period consumption elasticities

(b) Independent dad's consumption elasticities

Fig. 5: Uneducated dad: Elasticities for discount factors $\in [0.735, 0.98]$



(a) Per-capita Consumption : 20 joint-periods and 30 independent dad's periods

(b) Assets (all 50 years)

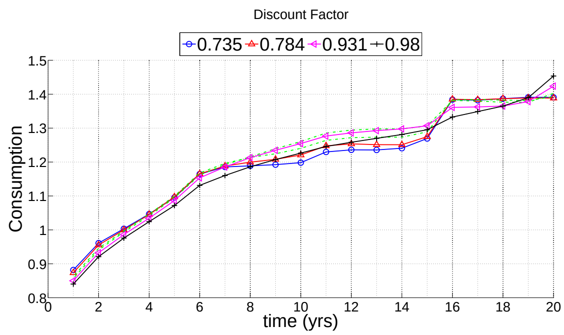
Fig. 6: Uneducated dad: Life-cycle plots for discount factors $\in [0.735, 0.98]$

Table 6: Uneducated dad: Impact of change in discount-factor from benchmark value of 0.98. $\beta \in [0.735, 0.98]$

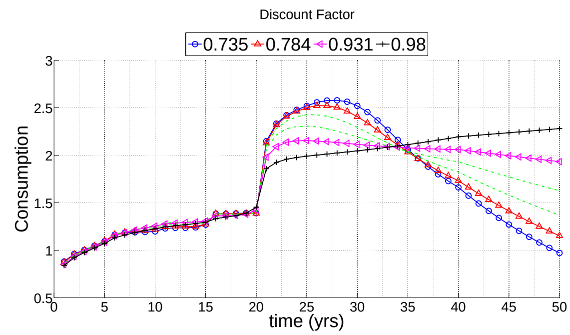
$\Delta\beta$	$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$	$\Delta C_{dad(21-40,41-50)}$	ΔBeq	$\Delta Edu(yrs)$	$\Delta Books(Tot)$	ΔH_{son}
-25 %	01.45, 01.05, 01.47, 26.78 %	30.17, -35.59 %	-51.23 %	0 %	0 %	00.00 %
-20 %	01.24, 00.90, 01.42, 26.51 %	25.18, -30.30 %	-43.70 %	0 %	0 %	00.00 %
-15 %	01.02, 00.77, 01.57, 25.99 %	19.05, -24.31 %	-35.11 %	0 %	0 %	00.00 %
-10 %	00.72, 00.39, 01.70, 24.39 %	12.01, -17.70 %	-25.45 %	0 %	0 %	00.00 %
-5 %	00.26, 00.42, 01.76, 16.32 %	05.39, -9.57 %	-13.78 %	0 %	0 %	00.00 %

$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$: % changes in per-capita consumptions (from benchmark) for time periods 1-5, 6-10, 11-15, 16-20.
$\Delta Edu(yrs)$, $\Delta Books(Tot)$: % changes in total years of education and books from benchmark values.

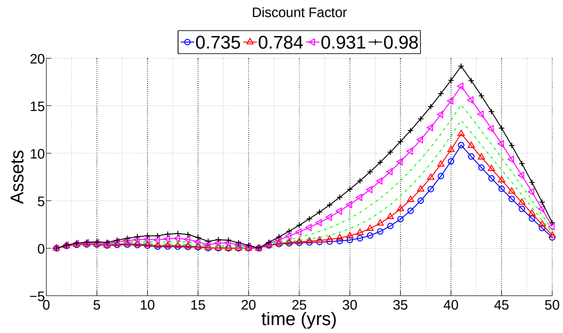
Matric Dad : Surprisingly, in the lower range of the spectrum, with increasing β, the educational investments begin to fall and hence, due to higher child-labour earnings, $C_{joint(6-10)}$ and $C_{joint(11-15)}$ increase as is reflected by the negative elasticities of human-capital investments and positive elasticities of consumptions (Figure 8). However, the trend gets reversed after a threshold where educational investments' and joint-period consumptions' elasticities switch trend. The independent dad's retired period consumption elasticity, owing to higher savings in the previous periods, is positive. $C_{dad(21-40)}$ elasticity is zero because consumption switches trend (Figure 8) at $t = 34$ so averaged over the period its net impact is marginal. Impact of discount rate on other consumptions is negligible.



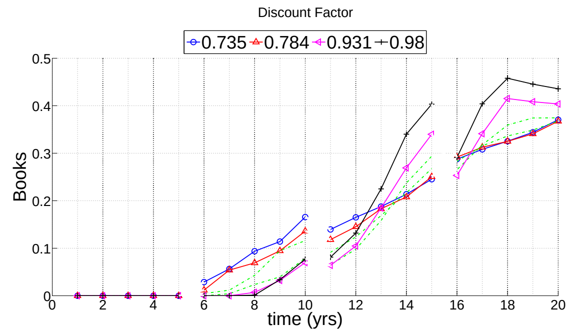
(a) Consumption (first 20 years)



(b) Consumption (all 50 years)

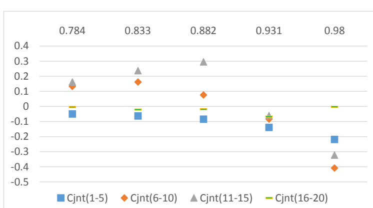


(c) Assets (all 50 years)

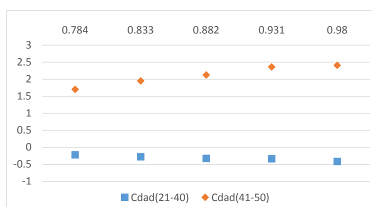


(d) Investment in Books

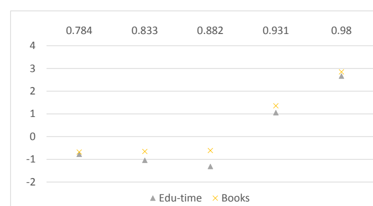
Fig. 7: Matric dad: Life-cycle plots for discount factors $\in [0.735, 0.98]$



(a) Joint-period consumption elasticities



(b) Independent dad's consumption elasticities



(c) Educational inputs' elasticities

Fig. 8: Matric dad: Elasticities for discount factors $\in [0.735, 0.98]$

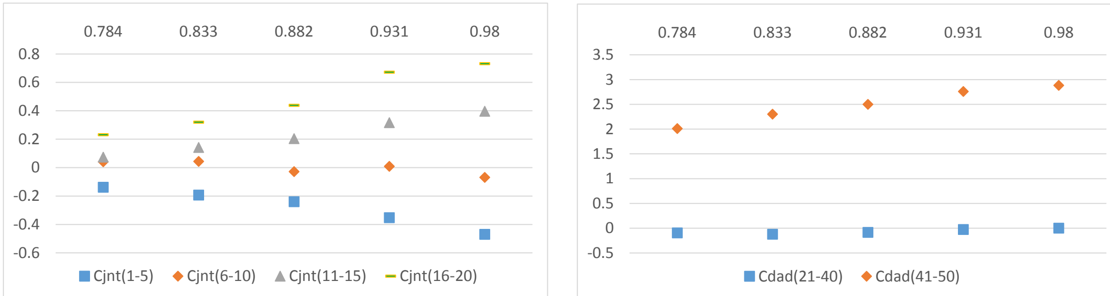
Table 7: Matric dad: Impact of change in discount-factor from benchmark value of 0.98. $\beta \in [0.735, 0.98]$

$\Delta\beta$	$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$	$\Delta C_{dad(21-40,41-50)}$	ΔBeq	$\Delta Edu(yrs)$	$\Delta Books(Tot)$	ΔH_{son}
-25 %	03.21, 00.31, -2.19, 00.66 %	09.72, -44.17 %	-57.39 %	01.27 %	-8.47 %	-5.85 %
-20 %	02.87, 01.20, -1.14, 00.63 %	08.10, -37.84 %	-49.48 %	-3.96 %	-12.60 %	-7.87 %
-15 %	02.46, 02.22, 00.33, 00.50 %	06.21, -30.26 %	-39.94 %	-10.22 %	-16.15 %	-9.77 %
-10 %	01.95, 02.67, 02.07, 00.39 %	04.17, -21.54 %	-28.71 %	-17.16 %	-19.15 %	-11.93 %
-5 %	01.17, 02.19, 01.72, 00.01 %	02.23, -11.25 %	-15.26 %	-12.31 %	-13.04 %	-8.14 %

$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$: % changes in per-capita consumptions (from benchmark) for time periods 1-5, 6-10, 11-15, 16-20.
$\Delta Edu(yrs)$, $\Delta Books(Tot)$: % changes in total years of education and books from benchmark values.

Graduate Dad : This is where the clear effect of discount rate gets exhibited. From Figure 10 it is evident that due to increasing patience, early period consumptions in the joint-phase ($C_{joint(1-5)}$) and independent dad's phase ($C_{dad(21-40)}$) have negative elasticities whereas later period consumptions in the two phases ($C_{joint(11-15)}, C_{joint(16-20)}$ and $C_{dad(41-50)}$) have positive elasticities (of 0.4, 0.7 and 2.8 respectively). $C_{dad(21-40)}$ elasticity is lower than expected as at $t = 34$

consumption switches trend which is midway through the independent dad's working period. Human-capital investments remain unaffected.



(a) Joint-period consumption elasticities

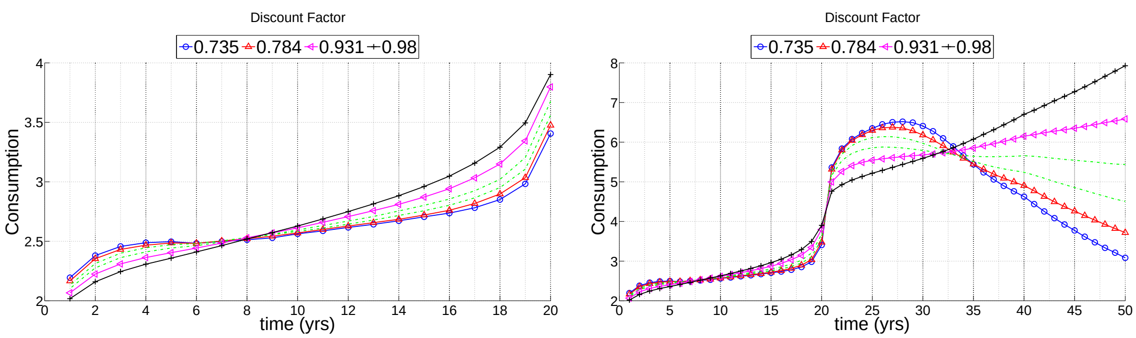
(b) Independent dad's consumption elasticities

Fig. 9: Graduate dad: Elasticities for discount factors $\in [0.735, 0.98]$

Table 8: Graduate dad: Impact of change in discount-factor from benchmark value of 0.98. $\beta \in [0.735, 0.98]$

$\Delta\beta$	$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$	$\Delta C_{dad(21-40,41-50)}$	ΔBeq	$\Delta Edu(yrs)$	$\Delta Books(Tot)$	ΔH_{son}
-25 %	08.38, -0.07, -6.18, -12.64 %	02.06, -49.40 %	-61.12 %	00.00 %	-0.00 %	-0.00 %
-20 %	07.38, 00.21, -5.72, -11.29 %	01.42, -42.62 %	-53.10 %	00.00 %	-0.00 %	-0.00 %
-15 %	06.08, 00.48, -4.88, -9.51 %	00.65, -34.36 %	-43.15 %	00.00 %	-0.00 %	-0.00 %
-10 %	04.59, 00.31, -3.74, -7.18 %	00.14, -24.71 %	-31.41 %	00.00 %	00.00 %	00.00 %
-5 %	02.53, 00.36, -2.05, -3.71 %	-0.00, -13.17 %	-16.91 %	00.00 %	00.00 %	00.00 %

$\Delta C_{jnt(1-5,6-10,11-15,16-20)}$: % changes in per-capita consumptions (from benchmark) for time periods 1-5, 6-10, 11-15, 16-20.
$\Delta Edu(yrs)$, $\Delta Books(Tot)$: % changes in total years of education and books from benchmark values.



(a) Consumption (first 20 years)

(b) Consumption (all 50 years)

Fig. 10: Graduate dad: Life-cycle plots for discount factors $\in [0.735, 0.98]$

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